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On Families of Bowen – Series-Like Maps for Surface Groups

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Abstract—We review some recent results on a class of maps, called Bowen – Series-like maps, obtained from a class of group presentations for surface groups. These maps are piecewise homeomorphisms of the circle with finitely many discontinuities. The topological entropy of each map in the class and its relationship with the growth function of the group presentation is discussed, as well as the computation of these invariants.

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This paper is dedicated to Alain Chenciner on the occasion of his 80th birthday

1. INTRODUCTION

This paper is a review of recent results [3] on a particular relationship between two classical theories: geometric groups and dynamical systems. The relationship we consider has a clear origin in two papers published in 1979 by Bowen [8] and Bowen – Series [9]. On one side, the groups are the well-known Fuchsian groups given by some specific action A on the hyperbolic plane $\mathbb{H}^2$. The idea is to associate, to such an action, a dynamical system given as a "piecewise Möbius" map $\Phi_A\colon S^1 \longrightarrow S^1$, where $S^1 = \partial\mathbb{H}^2$. The Bowen – Series idea has been revisited in [17] for surface groups $G = \pi_1(\Sigma)$, where Σ is a closed compact surface of negative Euler characteristic. In this approach, G is given by a finite presentation $P = \langle X|R\rangle$, where X is a symmetric generating set and R is a set of relations. We will consider *geometric* presentations, meaning that the Cayley two complex $\mathrm{Cay}^2(G, P)$ is planar. This implies, in particular, that the Cayley graph $\mathrm{Cay}^1(G, P)$ is a planar graph. Observe that the classical presentations of surface groups are geometric in this sense [21]. The basic idea is to associate a dynamical system to such a group presentation. In this case the dynamics is given by a piecewise homeomorphism of the circle $\Phi_P\colon S^1 \longrightarrow S^1$. In this approach, G is a Gromov hyperbolic group [16] and its (Gromov) boundary is $\partial G = S^1$.

The maps Φ_A and Φ_P constructed, respectively, in [8, 9] and [17] are particular realizations of this idea and are different in several aspects (when they can be compared, e. g., for the classical presentations). The Bowen – Series maps Φ_A are "piecewise Möbius", in particular, they

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